LESSON 10

Volume of Cylinders, Cones, and Spheres

Common Core State Standards

G-GMD.3 Use volume formulas for cylinders, pyramids, cones and spheres to solve problems.

G-MG.2 Apply concepts of density based on area and volume in modeling situations.

Mathematical Practices 1, 2, 4

CAREER SPOTLIGHT: Agricultural Engineer

Agricultural engineers use science and math with a focus on designing agricultural systems related to farming practices. This career draws from principles in other engineering fields, such as civil and mechanical engineering, and combines them with environmental sciences and biology to improve the production of plants and livestock.

- Discuss agricultural engineering with students by reading the Career Spotlight together.
- Find local colleges and universities with an agricultural engineering program to share with students.
- Research local companies that employ agricultural engineers and ask what they do for the companies.

Video: Agricultural Engineer

Have students watch this video, which describes the types of projects an agricultural engineer might work on.

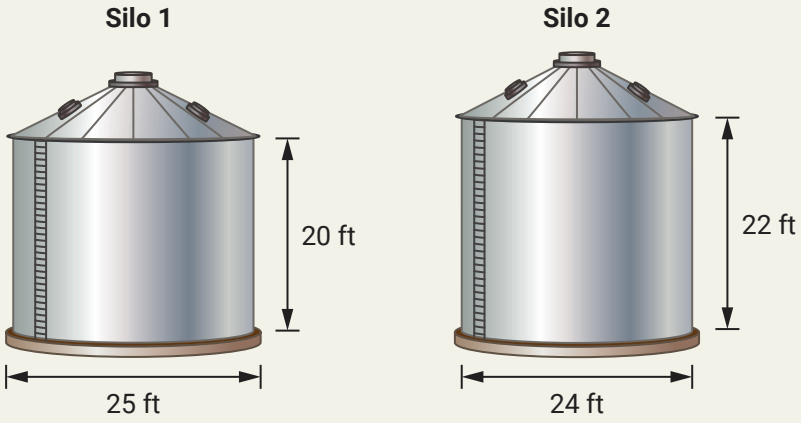
Lesson Objective

In this lesson, you will look at how an agricultural engineer uses the volume of solids when designing and evaluating structures and systems used in agricultural settings.

Teaching Support

1 Step Into the Career: Volume of Cylinders

An agricultural engineer is designing a farm storage system that will contain a silo for storing dried, shelled corn. The cylindrical part of the silo should store up to 400,000 pounds of corn. If the corn weighs 42 pounds per cubic foot, then which silo should be used?



Guiding Questions

- In Step 2, how can the number of pounds be found using a proportion?
- In Step 3, will a silo that has a diameter of 26 feet and a height of 20 feet hold the corn?

ENRICHMENT In this example, both silos can store 400,000 pounds of corn. Ask students to suggest what other criteria an agricultural engineer could consider in designing a silo. Discuss what considerations could be made about the amount of material needed to build the silos. Remind students that the lateral surface area of a cylinder can be determined by the formula $S = 2\pi rh$, where r is the radius and h is the height. Ask students to determine which silo has the greater lateral surface area.

TECHNOLOGY Challenge students to think about the dimensions of a cylinder that can hold 400,000 pounds of corn with the least amount of lateral surface area. Have students find the volume needed for 400,000 pounds of corn and then ask for an expression for the height h of a cylinder that can hold the corn in terms of radius r. Demonstrate using technology (by graphing or using a spreadsheet) how to determine the radius that results in the minimum lateral surface area.

On the Job: Apply Volume of Cylinders

Answers

1a. 314 ft^3

1b. about 2356 gallons

1c. 785 tilapia

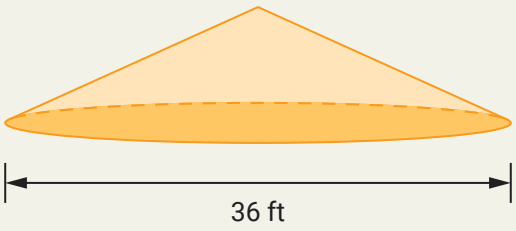
Use these questions to check students' understanding.

- In 1a, what formula did you used to find the volume? How did you determine the radius?
- In 1a, why is the volume of the tank in cubic feet an estimate?
- In 1b, how did you convert cubic feet to gallons?

2 Step Into the Career: Volume of Cones

An agricultural engineer designs an area for temporary storage of 2800 cubic feet of harvested wheat kernels. A cone-shaped pile of 2800 cubic feet of wheat kernels will have a diameter of 36 feet. At what minimum height above the ground should the end of the grain auger transporting the wheat be set so that it clears the pile?

The volume of the pile of wheat kernels is 2800 ft^3.



Students may not be familiar with a grain auger. A grain auger is a tube with a spiral shaft in the middle that transports grain. This is not to be confused with an auger that is a type of drilling device.

Guiding Questions

- Could the height be determined if only the volume of the pile was given and not the diameter?
- The height and diameter of a pile of wheat must remain in proportion. Suppose the volume of the wheat increases. Will the height of the pile be greater than or less than 8.25 feet? Explain.

ENRICHMENT The shape of a conical pile depends on the material. For wheat, the angle formed by a line from the vertex of the cone to the ground is about 25°. In this example, students can find this angle by calculating $\tan^{-1}\left(\frac{8.25}{18}\right) \approx 25°$. If the grain for this pile is barley, the angle is about 28°. Ask students to determine the height and volume of a pile of barley with diameter 36 feet.

On the Job: Apply Volume of Cones

Answers

2a. 15,240 cm^3

2b. for 1.5: 1292 cm^3/s, for 2.0: 3717 cm^3/s, for 2.5: 7620 cm^3/s

2c. setting 2.0

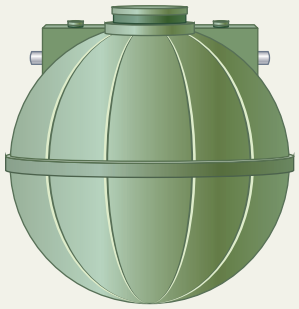
Use these questions to check students' understanding.

- In 2a, what volume formula did you use?
- In 2a, is the estimate for the volume of the seeds less than or greater than the actual amount of seeds? How do you know?
- In 2b, how did you find each flow rate?

3 Step Into the Career: Volume of Spheres

An irrigation system is being designed to contain underground water storage tanks like the one shown. The manufacturer says that the tank needs to be at least 25% full at all times to prevent any movement or distortion of the tank.

If the diameter of the spherical storage space of the tank is 70 inches, then what is the minimum number of gallons of water that need to be in the tank at all times? (Use 231 $in.^3 = 1$ gal.)



Guiding Questions

- In Step 2, how can the number of gallons be found using a proportion?
- In Step 3, suppose the tank needs to be at least 30% full. What is the minimum number of gallons that should be in the tank at all times?

On the Job: Apply Volume of Spheres

Answers

3a. 10.3 cm^3, 18.8 cm^3

3b. about 0.87 g/cm^3, about 0.80 g/cm^3

3c. yes

Use these questions to check student's understanding.

- In 3a, what volume formula did you use?
- In 3a, how did you find the radius?
- In 3b, how can the given units for the density help you find the densities?

Career Spotlight: Practice

Solution Steps for Exercises 4–7

These steps will help guide students in solving these practice exercises.

Exercise 4

Answer

4a. 8058 gal

4b. no; Sample explanation: 8058 gallons of water with a density of 8.3 pounds per gallon will weigh $8058 \cdot 8.3 = 66{,}881.4$ pounds. Since $66{,}881.4 < 70{,}000$, a full tank of water will not exceed the weight limit.

Solution Steps

- Use the volume formula for a cylinder to find the volume with $r = 3$ ft and $h = 38$ feet. (1074.4 ft^3)
- Multiply the volume by 7.5 to find the capacity in gallons. (about 8058 gal)
- Multiply the capacity in gallons by the density of water (8.3 lb/gal) to find the weight of water. (66,881.4 lb)
- Compare the weight of water to the weight limit. (The weight of water is under the weight limit.)

Exercise 5

Answer

5. yellow: 89%, sweet: 93%; The sweet onion needs special handling.

Solution Steps

- Use the volume formula of a sphere to find the volume of each onion. (180 cm^3, 113 cm^3)
- Write the volume of each onion in milliliters. (180 mL, 113mL)
- Find the percent of water in each onion. (89%, 93%)
- Determine whether each percent of water content is greater than 90%. (The sweet onion has a percent of water content greater than 90%.)

Exercise 6

Answer

6. about 44 minutes

Devise a Plan

Possible plan:

Step 1: Find the volume of the tank that does not contain water.

Step 2: Convert the volume in cubic feet to gallons.

Step 3: Use the rate to find the time.

Solution Steps

- Use the volume formula for a cylinder to find the volume of the tank that does not contain water. (471.2 ft^3)
- Multiply the volume of the tank that is not water by 7.5 gal/ft^3 to convert to gallons. (3534 gal)
- Divide the number of gallons left to fill by 80 gal/min to find time to fill tank in minutes. (44 min)

Exercise 7

Answer

7. about 173,642 lb

Solution Steps

- Use the volume formula for a cone to find the volume of each pile. (2850.05 ft^3, 844.46 ft^3)
- Find the sum of the volumes. (3694.51 ft^3)
- Multiply the sum by 47 lb/ft^3 to find the total weight of the piles. (173,642 lb)

Career Spotlight: Check

Tips for Completing Technology Enhanced Items 8–12

These tips will help students in solving these and similar assessment items.

Exercise 8

Answer

8. b. 7.1, a. 6, c. 42.6

Tip Encourage students to check their answers for reasonableness by reading the entire problem after they have chosen their answers. For example, the product of the numbers selected for the first and second blanks should equal the number selected for the third blank.

Exercise 9

Answer

9. A

Tip Encourage students to examine the answer choices after reading the problem to eliminate choices that can easily be identified as incorrect. For example, the diagram shows that the sediment does not appear to be 98% of the water, so choice D can be eliminated.

Exercise 10

Answer

10. 32

Tip Encourage students to write their answer down on paper before entering the answer using technology. Once entered, they can check their answer against the one written on paper. Tell students to be aware of dropped digits and transposed digits.

Exercise 11

Answer

11. b, c

Tip Encourage students to determine and find the information that is needed to select the true statements. For example, students should find the radius, volume, and weight of the pile of corn and the number of truckloads needed to transport the corn in order to select the true statements.

Exercise 12

Answer

12. Sample 1: 57%, Sample 2: 50%, Sample 3: 53%

Tip Encourage students to work efficiently when answering a matching problem. For example, by process of elimination, students only need to find the percent of water content for two samples.